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FVM for Nonlinear Soil Stress analysis involving Pore Pressure Coupling

Tian Tang*[1], Ole Hededal[1], Philip Cardiff[2], Johan Roenby[3]

[1]Technical University of Denmark, [2]University College Dublin, [3]DHI

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Outline

Abbreviations

Outline

Why we started with this research topic?

- Offshore structure and foundation failures due to seabed instability (liquefaction) are observed.
- Integrated numerical modelling of seabed-wave-structure interaction is demanding.
- OpenFOAM as an open source FVM library facilitates the customer solver developments.

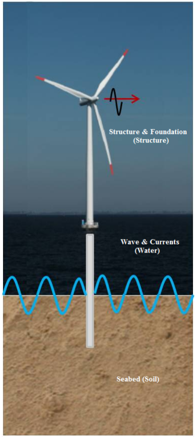

Figure: An illustration of wave-seabed-structure interaction

Our Goals

- Current goal: Development of an efficient soil solver with plastic soil deformation and pore pressure coupling.
- Future goal: Multiphysics modeling by combining the developed soil solver with existing fluid and structure solvers

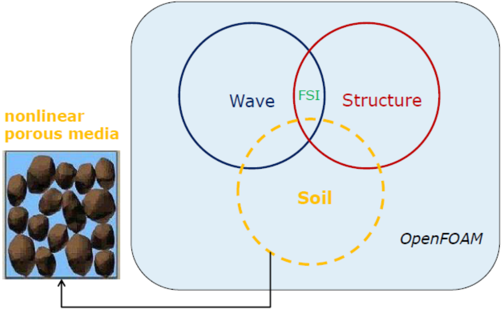

Figure: The multiphysics solver structure using OpenFOAM

Outline

Soil Mathematical Model

Governing Equations:

- Total momentum balance for the soil mixture (steady-state)

$$\nabla \cdot \boldsymbol{\sigma} - \nabla p = 0$$

- Storage equation for pore fluid flow

$$\frac{n}{K'}\frac{\partial p}{\partial t} - \frac{k}{\gamma_w}\nabla^2 p + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{u}) = 0$$

$\boldsymbol{\sigma}$: soil effective stress, p: pore fluid pressure,
$\mathbf{u}$: soil skeleton displacement,
n, K': soil porosity and pore fluid bulk modulus,
k, γ_w: soil permeability and water specific weight.

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nonlinear consitutive relation & displacement-pressure coupling

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$\mathbf{u}$: soil skeleton displacement,
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Soil Mathematical Model

Constitutive relations:

- Linear elasticity

$$d\boldsymbol{\sigma} = \mathbf{C}^e : (d\varepsilon - d\varepsilon^p), \quad d\varepsilon = \frac{1}{2}\left\{\nabla(d\mathbf{u}) + [\nabla(d\mathbf{u})]^{\mathsf{T}}\right\}$$

- Mohr-Coulomb perfect plasticity

$$\text{Yield surface} \quad f = (\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3)\sin\varphi - 2c\cos\varphi$$

$$\text{Plastic potential} \quad g = (\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3)\sin\psi$$

$$\text{Flow rule} \quad d\varepsilon^p = d\Lambda \cdot \frac{\partial g}{\partial \boldsymbol{\sigma}}, \quad d\Lambda = \frac{\left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\right)^{\mathsf{T}} \mathbf{C}^e d\varepsilon}{\left(\frac{\partial f}{\partial \boldsymbol{\sigma}}\right)^{\mathsf{T}} \mathbf{C}^e \frac{\partial g}{\partial \boldsymbol{\sigma}}}$$

σ_1, σ_3: maximum and minimum principal stress,
φ, c, ψ:soil friction angle, cohesion, and dilation angle,
C^e: linear elastic stiffness tensor.

Discretization & Solution procedure

- Cell-centered finite volume discretization
- Global solution procedure:
  1. Partitioned (segregated) approach

```
fvScalarMatrix pEqn
            (
                fvm::ddt(p) == fvm::laplacian(Dp, p) - fvc::div(fvc::ddt(Dp2,U))
            );
```

```
fvVectorMatrix dUEqn
            (
                fvm::laplacian(2.0*mu + lambda, dU, "laplacian(dU)")
              ==
                - divDsigmaExp
                + fvc::div(2.0*mu*(mesh.Sf() & fvc::interpolate(dEpsP)))
                + fvc::div(lambda*(mesh.Sf() & I*fvc::interpolate(tr(dEpsP))))
                + fvc::grad(dp)
            );
```

Discretization & Solution procedure

2. Fixed Point iteration + Underrelaxation

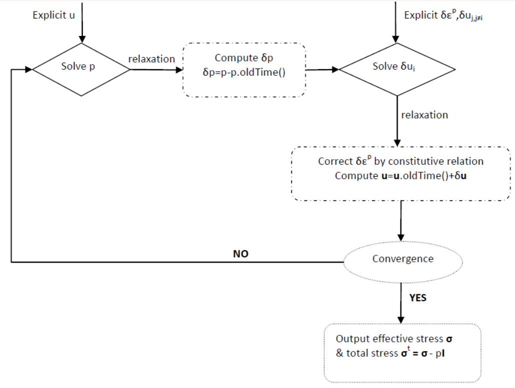


Figure: The iterative solution strategy of `nonLinearBiotFoam` in OpenFOAM

Discretization & Solution procedure

- Local stress update

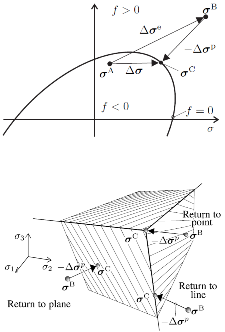


Figure: Illustration of return mapping

Stress return algorithm

INPUT: $d\mathbf{u}$, displacement increments
$\boldsymbol{\sigma}^A$, initial/old-time stress

1. Compute the elastic trial stress $\boldsymbol{\sigma}^B$ by:
   $\boldsymbol{\sigma}^B = \boldsymbol{\sigma}^A + \{\mu\nabla(d\mathbf{u}) + \mu[\nabla(d\mathbf{u})]^{\mathrm{T}} + \lambda\mathbf{I}\mathrm{tr}[(d\mathbf{u})]\}$
2. Transform $\boldsymbol{\sigma}^B$ into principal space as $\boldsymbol{\sigma}^B_{prin}$. Store the principal directions.
3. Evaluate the yield function $f(\boldsymbol{\sigma}^B_{prin})$:
   if $f < 0$, EXIT, $\boldsymbol{\sigma}^C = \boldsymbol{\sigma}^B$, $d\boldsymbol{\varepsilon}^p = 0$
   if $f \geq 0$, CONTINUE
4. Determine the right stress return type.
   Obtain the principal plastic corrector stress $\boldsymbol{\sigma}^C_{prin}$.
5. Reuse the preserved principal directions and transform $\boldsymbol{\sigma}^C_{prin}$ back to $\boldsymbol{\sigma}^C$
6. Calculate the plastic strain increment $d\boldsymbol{\varepsilon}^p$

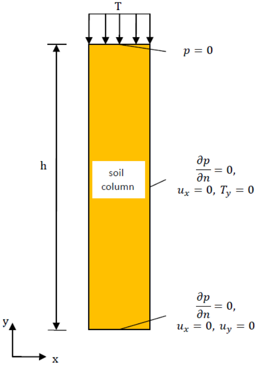
OUTPUT: $\boldsymbol{\sigma}^C$, $d\boldsymbol{\varepsilon}^p$

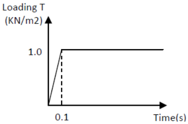
Test cases

- Elastic consolidation test
- Drained triaxial compression test
- Elasto-plastic consolidation test

Test cases: Elastic consolidation test

A saturated soil column subjected to a surface step loading:





Soil parameters:

$$E = 3.0 \times 10^7 N/m^2$$
$$\nu = 0.2$$
$$k = 0.0001 m/s$$
$$n = 0.3$$
$$\gamma_w = 1.0 \times 10^4 N/m^2$$
$$K_w = 2.1 \times 10^9 N/m^2, S_r = 0.99$$
$$h = 30m$$

Figure: Case definition

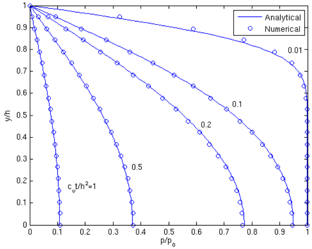
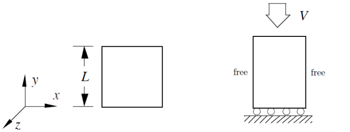


Figure: Pore pressure distribution along the depth after different consolidation time

Test cases: Drained triaxial compression test

A drained soil cube compressed by constant strain rate:



(a) solution domain (b) boundary conditions

Soil parameters:
$E = 9.5 \times 10^7 N/m^2$, $\nu = 0.185$
$\varphi = 40^\circ$, $c = 5 \times 10^3 N/m^2$, $\psi = 10^\circ$
$L = 0.1m$

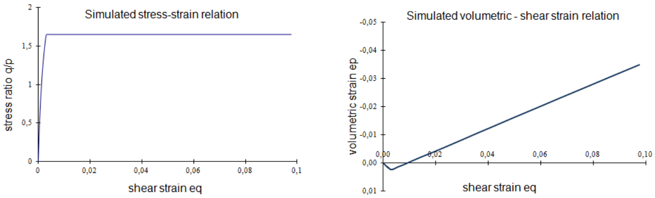
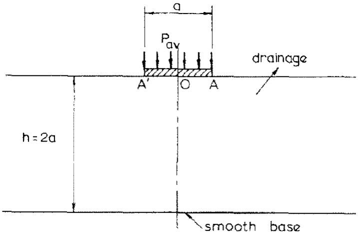


Figure: Simulated elastic perfect-plastic soil response

Test cases: Elasto-plastic consolidation test

A layer of saturated soil loaded by a strip footing with different loading rate:



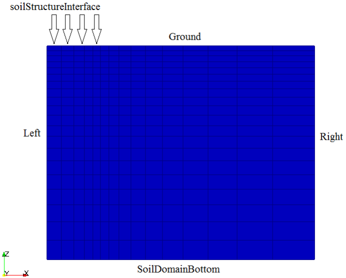


Figure: A sketch of the case geometry (left); OpenFOAM mesh and boundary conditions (right)

Test cases: Elasto-plastic consolidation test

A layer of saturated soil loaded by a strip footing with different loading rate:

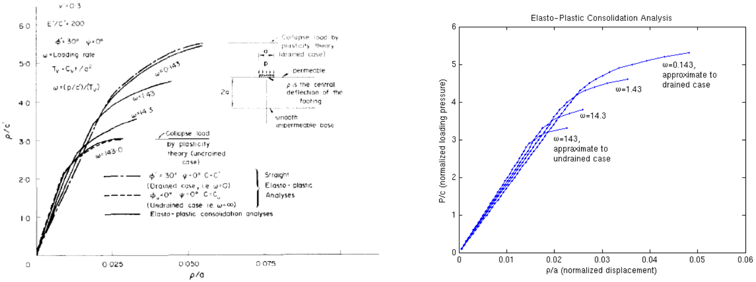


Figure: FEM results by Small et al. (left); FVM results by `nonLinearBiotFoam` (right).

Test cases: Elasto-plastic consolidation test

Animations: Pore pressure variation + Yielding zone

(Loading Video...)

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(displacement exaggerated by a factor of 10)

Outline

Modeling more soil features: Large deformation

- Linear elastic stress-strain + Nonlinear strain-displacement relation (Total Lagrangian format):

$$\delta \mathbf{S} = 2\mu \delta \mathbf{E} + \lambda \mathrm{tr}(\delta \mathbf{E})\mathbf{I}$$

$$\delta \mathbf{E} = \frac{1}{2}[\nabla(\delta \mathbf{u}) + \nabla(\delta \mathbf{u})^{\mathrm{T}} + \nabla(\delta \mathbf{u}) \cdot \nabla \mathbf{u}^{\mathrm{T}} + \nabla \mathbf{u} \cdot \nabla(\delta \mathbf{u})^{\mathrm{T}} + \nabla(\delta \mathbf{u}) \cdot \nabla(\delta \mathbf{u})^{\mathrm{T}}]$$

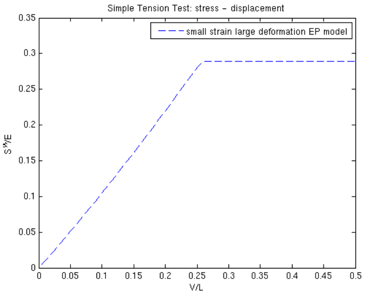
- All the nonlinear terms treated explicit + Fixed point iteration:

$$\underbrace{(2\mu + \lambda)\nabla^2(\delta \mathbf{u})}_{implicit} + \underbrace{\nabla \cdot \{-(\mu + \lambda)\nabla(\delta \mathbf{u}) + \mu[\nabla(\delta \mathbf{u})^{\mathrm{T}} + \nabla(\delta \mathbf{u}) \cdot \nabla \mathbf{u}^{\mathrm{T}} + \nabla \mathbf{u} \cdot \nabla(\delta \mathbf{u})^{\mathrm{T}} + \nabla(\delta \mathbf{u}) \cdot \nabla(\delta \mathbf{u})^{\mathrm{T}}] + \lambda \mathrm{tr}(\delta \mathbf{E})\}}_{explicit}$$
$$+ \underbrace{\nabla \cdot [\delta \mathbf{S} \cdot \nabla \mathbf{u} + (\mathbf{S} + \delta \mathbf{S}) \cdot \nabla(\delta \mathbf{u})]}_{explicit} = \mathbf{0}$$

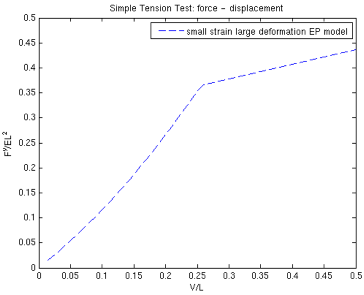
- Extend to small strain large displacement elasto-plastic solver (similar solution procedure to small strain small displacement EP solver)

Modeling more soil features: Large deformation

- Verification: a simple tension test



(a)



(b)

Figure: a) Resulting stress-displacement relationships from `soilEpTLFoam` prediction; b) Resulting force-displacement relationships from `soilEpTLFoam` prediction

Modeling more soil features: Anisotropy

Soil deposits are inherently anisotropic due to the process of sedimentaton followed by predominantly one-dimensional consolidation. Soil anisotropy is used in reference to soil structure, soil strength, and soil permeability changes with direction of measurement.

- Assumption: cross-anisotropic soil

$$\mathbf{C}^e = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & (C_{11} - C_{12})/2 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_1 & 0 \\ 0 & 0 & k_3 \end{pmatrix}$$

$\mathbf{C}^e$: cross-anisotropic elastic stiffness tensor, $\mathbf{k}$: permeability coefficient tensor.

Modeling more soil features: Anisotropy

- Implementation:

$$\underbrace{\nabla\cdot(\mathbf{K}\cdot\nabla(d\mathbf{u}))}_{Implicit}+\underbrace{\nabla\cdot(\mathbf{C}:d\varepsilon)-\nabla\cdot(\mathbf{K}\cdot\nabla(d\mathbf{u}))-\nabla p}_{explicit}=0$$

$$\underbrace{\frac{n}{K'}\frac{\partial p}{\partial t}-\frac{1}{\gamma_w}\nabla\cdot(\mathbf{k}\cdot\nabla p)}_{Implicit}+\underbrace{\frac{\partial}{\partial t}(\nabla\cdot\mathbf{u})}_{explicit}=0$$

- Tested by a case of standing wave induced cross-anisotropic seabed response.

Modeling more soil features: Anisotropy

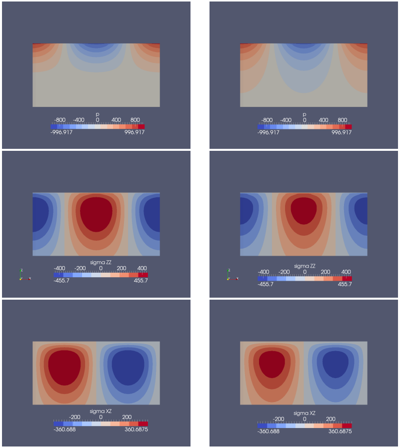


Figure: Standing-wave induced anisotropic and isotropic soil response. Isotropy(right): $E = 10^7 Pa, \nu = 0.3$, cross-anisotropy(left): $E_z = 10^7 Pa, \nu_{xx} = \nu_{zx} = 0.3, n = m = 0.6, k_x = k_z = 10^{-4} m/s$.

Convergence consideration

Fixed point iteration method in FVM has both advantages and drawbacks:

- No need to form and update the Jacobian matrix. ☺
- Create diagonally dominant sparse matrices ideally suited for iterative solver. ☺
- No convergence for some highly nonlinear and strong coupling problems. If combined with fixed underrelaxation, slow convergence. ☹

Seek for adaptive underrelaxation → Aitken's method

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Seek for adaptive underrelaxation $\rightarrow$ Aitken's method

Convergence consideration: Aitken's method

Aitken's method is most useful for accelerating a linear convergent sequence:

$$x^{i+1} = \tilde{x}^{i+1} + \theta^{i+1} r^i$$

$$r^i = \tilde{x}^{i+1} - x^i$$

$$\theta^{i+1} = -\theta^i \frac{r^{i-1}\left(r^i - r^{i-1}\right)}{\left(r^i - r^{i-1}\right)\left(r^i - r^{i-1}\right)}$$

where, x is the solving variable (or variable vector). The tilde sign denotes the solved value before underrelaxation.

Apply to a large deformation elastoplastic simple tension test case:

	Number of outer iterations (plastic step)	total CPU time (s)
Fixed under-relaxation	618	38.34
Aitken's relaxation	264	15.79

Outline

Summary & Future works

Summary:

- FVM soil solver has been developed using OpenFOAM, it has the following features:
  - elasto-plastic soil deformations
  - pore pressure coupling
- The nonlinearity and coupling in the equations are tackled by partitioned approach and fixed point iteration.
- Large deformation and soil anisotropy added to the soil solver.
- Aitken's method is applied for convergence acceleration.

Next step:

- Implementaion of advanced soil solver based on critical state and cyclic plasticity.
- Further convergence improvement.

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Thank you for your attention!